NAG Toolbox for MATLAB

f08pp

1 Purpose

f08pp computes the eigenvalues, the Schur form T, and, optionally, the matrix of Schur vectors Z for an n by n complex nonsymmetric matrix A.

2 Syntax

```
[a, sdim, w, vs, rconde, rcondv, info] = f08pp(jobvs, sort, select, sense, a, 'n', n)
```

3 Description

The Schur factorization of A is given by

$$A = ZTZ^{\mathrm{H}},$$

where Z is orthogonal, the matrix of Schur vectors, and T is upper triangular.

Optionally, f08pp also orders the eigenvalues on the diagonal of the Schur form so that selected eigenvalues are at the top left; computes a reciprocal condition number for the average of the selected eigenvalues (**rconde**); and computes a reciprocal condition number for the right invariant subspace corresponding to the selected eigenvalues (**rcondv**). The leading columns of Z form an orthonormal basis for this invariant subspace.

For further explanation of the reciprocal condition numbers **rconde** and **rcondv**, see Section 4.8 of Anderson *et al.* 1999 (where these quantities are called s and sep respectively).

A complex matrix is in Schur form if it is upper triangular.

4 References

Anderson E, Bai Z, Bischof C, Blackford S, Demmel J, Dongarra J J, Du Croz J J, Greenbaum A, Hammarling S, McKenney A and Sorensen D 1999 *LAPACK Users' Guide* (3rd Edition) SIAM, Philadelphia URL: http://www.netlib.org/lapack/lug

Golub G H and Van Loan C F 1996 *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

5.1 Compulsory Input Parameters

1: **jobvs – string**

If **jobvs** = 'N', Schur vectors are not computed.

If **jobvs** = 'V', Schur vectors are computed.

Constraint: **jobvs** = 'N' or 'V'.

2: **sort – string**

Specifies whether or not to order the eigenvalues on the diagonal of the Schur form.

sort = 'N'

Eigenvalues are not ordered.

sort = 'S'

Eigenvalues are ordered (see user-supplied logical function **select**).

Constraint: **sort** = 'N' or 'S'.

3: **select – string containing name of m-file**

If **sort** = 'S', **select** is used to select eigenvalues to sort to the top left of the Schur form.

If **sort** = 'N', **select** is not referenced and f08pp may be called with the string `'f08pnz'`.

Its specification is:

```
[result] = select(w)
```

Input Parameters

1: **w – complex scalar**

The eigenvalue $\mathbf{w}(j)$ is selected if **select**$(\mathbf{w}(j))$ is **true**.

Output Parameters

1: **result – logical scalar**

The result of the function.

4: **sense – string**

Determines which reciprocal condition numbers are computed.

sense = 'N'

None are computed.

sense = 'E'

Computed for average of selected eigenvalues only.

sense = 'V'

Computed for selected right invariant subspace only.

sense = 'B'

Computed for both.

If **sense** = 'E', 'V' or 'B', **sort** must equal 'S'.

Constraint: **sense** = 'N', 'E', 'V' or 'B'.

5: **a**(**lda**,$*$) **– complex array**

The first dimension of the array **a** must be at least $\max(1, \mathbf{n})$

The second dimension of the array must be at least $\max(1, \mathbf{n})$

The n by n matrix A.

5.2 Optional Input Parameters

1: **n – int32 scalar**

Default: The first dimension of the array **a** and the second dimension of the array **a**. (An error is raised if these dimensions are not equal.)

n, the order of the matrix A.

Constraint: $\mathbf{n} \geq 0$.

5.3 Input Parameters Omitted from the MATLAB Interface

lda, ldvs, work, lwork, rwork, bwork

5.4 Output Parameters

1: **a(lda,∗) – complex array**

The first dimension of the array **a** must be at least $\max(1, \mathbf{n})$

The second dimension of the array must be at least $\max(1, \mathbf{n})$

a contains its Schur form T.

2: **sdim – int32 scalar**

If **sort** = 'N', **sdim** = 0.

If **sort** = 'S', **sdim** = number of eigenvalues for which user-supplied logical function **select** is **true**.

3: **w(∗) – complex array**

Note: the dimension of the array **w** must be at least $\max(1, \mathbf{n})$.

Contains the computed eigenvalues, in the same order that they appear on the diagonal of the output Schur form T.

4: **vs(ldvs,∗) – complex array**

The first dimension, **ldvs**, of the array **vs** must satisfy

if **jobvs** = 'V', **ldvs** $\geq \max(1, \mathbf{n})$;
ldvs ≥ 1 otherwise.

The second dimension of the array must be at least $\max(1, \mathbf{n})$ if **jobvs** = 'V', and at least 1 otherwise

If **jobvs** = 'V', **vs** contains the unitary matrix Z of Schur vectors.

If **jobvs** = 'N', **vs** is not referenced.

5: **rconde – double scalar**

If **sense** = 'E' or 'B', contains the reciprocal condition number for the average of the selected eigenvalues.

If **sense** = 'N' or 'V', **rconde** is not referenced.

6: **rcondv – double scalar**

If **sense** = 'V' or 'B', **rcondv** contains the reciprocal condition number for the selected right invariant subspace.

If **sense** = 'N' or 'E', **rcondv** is not referenced.

7: **info – int32 scalar**

info = 0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

info $= -i$

If **info** $= -i$, parameter i had an illegal value on entry. The parameters are numbered as follows:

1: **jobvs**, 2: **sort**, 3: **select**, 4: **sense**, 5: **n**, 6: **a**, 7: **lda**, 8: **sdim**, 9: **w**, 10: **vs**, 11: **ldvs**, 12: **rconde**, 13: **rcondv**, 14: **work**, 15: **lwork**, 16: **rwork**, 17: **bwork**, 18: **info**.

It is possible that **info** refers to a parameter that is omitted from the MATLAB interface. This usually indicates that an error in one of the other input parameters has caused an incorrect value to be inferred.

info $= 1$ to N

If **info** $= i$ and $i \leq$ **n**, the QR algorithm failed to compute all the eigenvalues

info $= N + 1$

The eigenvalues could not be reordered because some eigenvalues were too close to separate (the problem is very ill-conditioned).

info $= N + 2$

After reordering, roundoff changed values of some complex eigenvalues so that leading eigenvalues in the Schur form no longer satisfy **select** $=$ **true**. This could also be caused by underflow due to scaling.

7 Accuracy

The computed Schur factorization satisfies

$$A + E = ZTZ^{\mathrm{T}},$$

where

$$\|E\|_2 = O(\epsilon)\|A\|_2,$$

and ϵ is the ***machine precision***. See Section 4.8 of Anderson *et al.* 1999 for further details.

8 Further Comments

The total number of floating-point operations is proportional to n^3.

The real analogue of this function is f08pb.

9 Example

f08pp_select.m

```
function [result] = select(w)
  if (real(w) > 0)
    result = true;
  else
    result = false;
  end;
```

```
jobvs = 'Vectors (Schur)';
sort = 'Sort';
sense = 'Both reciprocal condition numbers';
```

```
a = [complex(-3.97, -5.04), complex(-4.11, +3.7), complex(-0.34, +1.01),
complex(1.29, -0.86);
       complex(0.34, -1.5), complex(1.52, -0.43), complex(1.88, -5.38),
complex(3.36, +0.65);
       complex(3.31, -3.85), complex(2.5, +3.45), complex(0.88, -1.08),
complex(0.64, -1.48);
      complex(-1.1, +0.82), complex(1.81, -1.59), complex(3.25, +1.33),
complex(1.57, -3.44)];
[aOut, sdim, w, vs, rconde, rcondv, info] = f08pp(jobvs, sort,
'f08pp_select', sense, a)
```

```
aOut =
    7.9982 - 0.9964i    0.9241 - 0.5720i   -0.3369 + 0.2728i   -0.5509 +
0.4531i
         0               3.0023 - 3.9998i    0.6598 - 0.6469i    0.0621 -
0.3566i
         0                    0             -6.0004 - 6.9998i   -0.4934 -
0.1212i
         0                    0                   0             -5.0000 +
2.0060i
sdim =
          2
w =
   7.9982 - 0.9964i
   3.0023 - 3.9998i
  -6.0004 - 6.9998i
  -5.0000 + 2.0060i
vs =
    0.1178 - 0.2954i   -0.1767 + 0.0785i   -0.7858 + 0.3324i    0.2072 -
0.3009i
   -0.6792 + 0.1345i    0.1328 - 0.1243i   -0.0958 - 0.3307i    0.0242 -
0.6069i
   -0.4224 - 0.4219i   -0.3726 + 0.1758i   -0.1789 - 0.2291i   -0.4825 +
0.3956i
   -0.2428 + 0.0630i   -0.4696 - 0.7343i    0.0741 + 0.2519i    0.2653 +
0.1949i
rconde =
    0.9928
rcondv =
    8.4118
info =
          0
```